Théorème

- Si $(u_n)_{n\in\mathbb{N}}$ est croissante majorée par M, alors $(u_n)_{n\in\mathbb{N}}$ converge et sa limite vérifie $\lim u_n \leq M$.
- Si $(u_n)_{n\in\mathbb{N}}$ est croissante non majorée, alors $\lim u_n = +\infty$.

On dispose de résultats analogues pour les suites décroissantes.

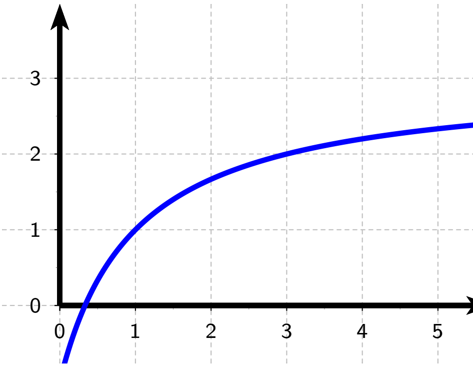
ex 28

ex 28

- $f(x) = \frac{3x-1}{x+1}$
-
-
-

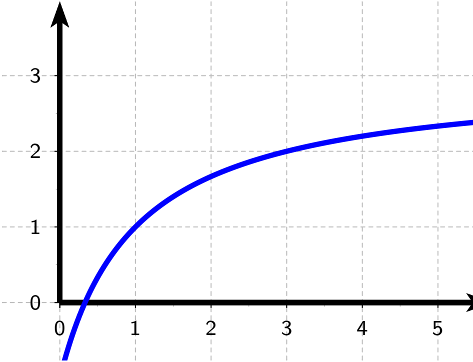
ex 28

- $f(x) = \frac{3x-1}{x+1}$
-
-
-



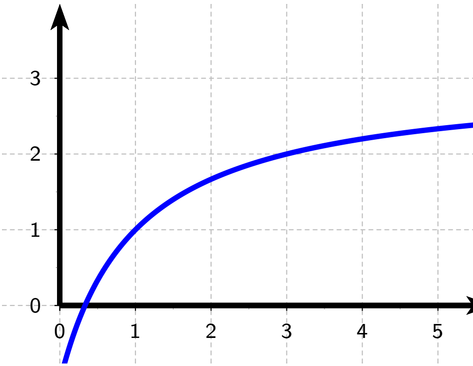
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) \end{cases}$
-
-



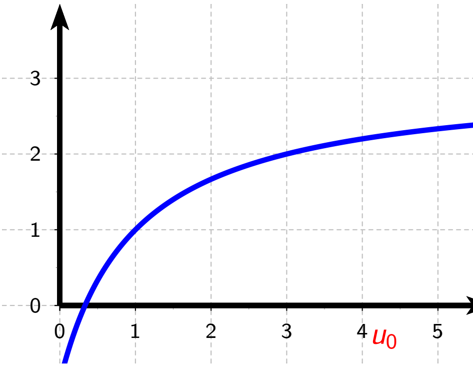
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n-1}{u_n+1} \end{cases}$
-
-



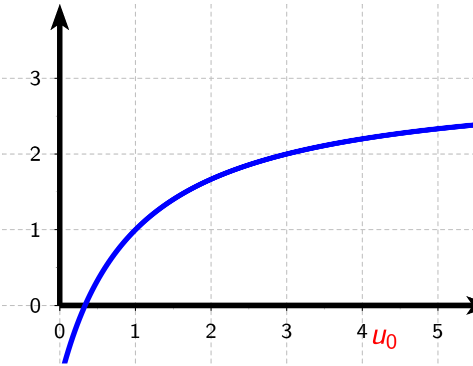
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
-
-

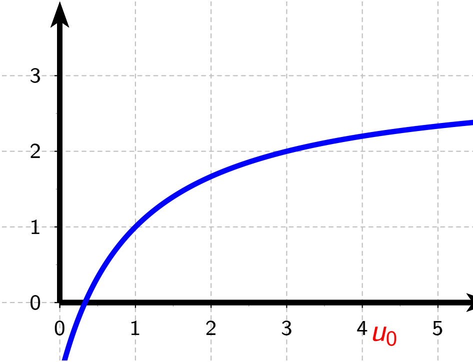


ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 =$
-

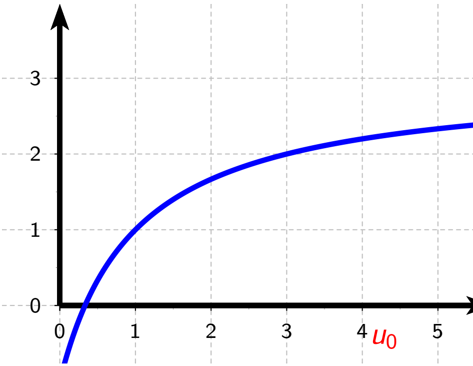


- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times4-1}{4+1}$
-



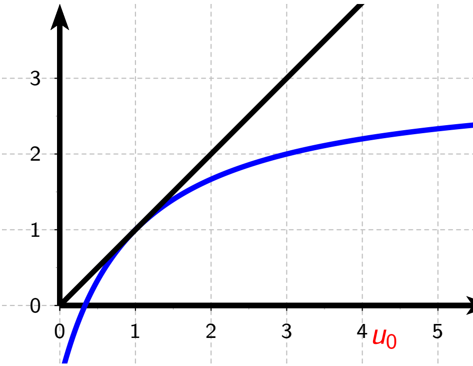
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times4-1}{4+1} = 2,2$
-



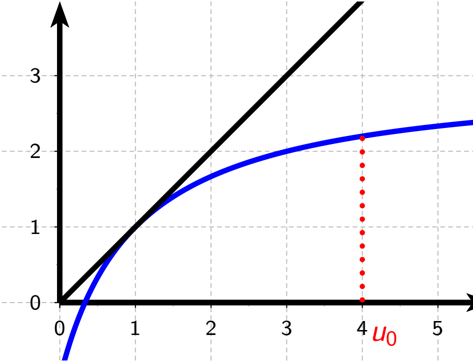
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times 4-1}{4+1} = 2,2$
-



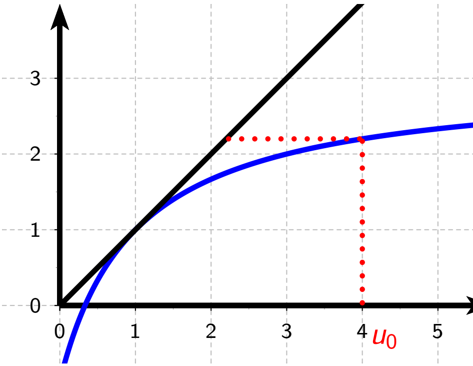
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times4-1}{4+1} = 2,2$
-



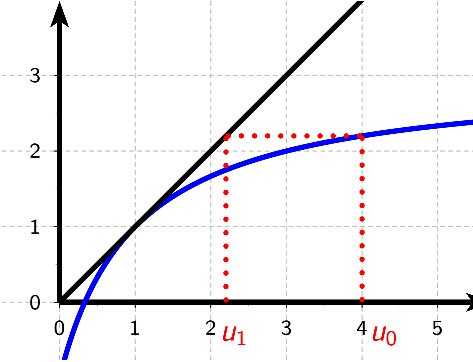
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times4-1}{4+1} = 2,2$
-



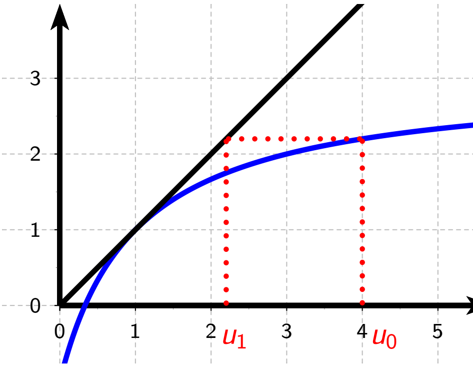
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times 4-1}{4+1} = 2,2$
-



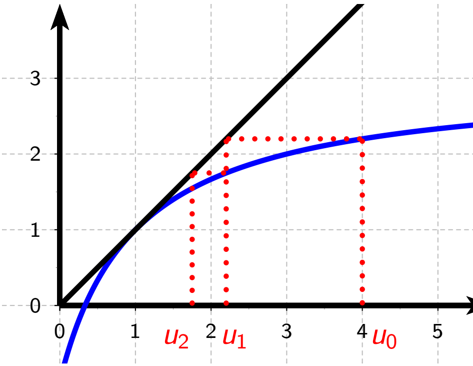
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times 4-1}{4+1} = 2,2$
- $u_2 = \frac{3\times 2,2-1}{2,2+1} = 1,75$



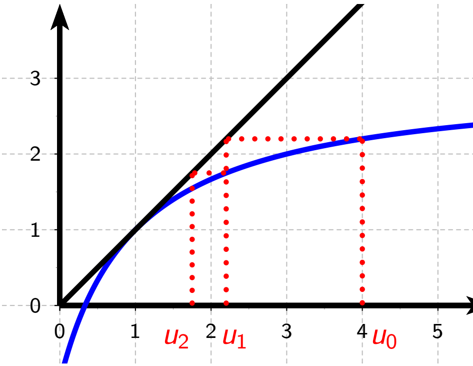
ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times4-1}{4+1} = 2,2$
- $u_2 = \frac{3\times2{,}2-1}{2{,}2+1} = 1,75$



ex 28

- $f(x) = \frac{3x-1}{x+1}$
- $\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n-1}{u_n+1} \end{cases}$
- $u_1 = \frac{3\times4-1}{4+1} = 2,2$
- $u_2 = \frac{3\times2{,}2-1}{2{,}2+1} = 1,75$



$\forall n \in \mathbb{N},\ 1 \leq u_{n+1} \leq u_n \leq 4$

ex 28

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

-
-
-

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

ex 28

$$\begin{cases} u_0 &= 4 \\ u_{n+1} &= f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1		4
$f(x)$			2.2
		↗	
	1		

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1		4
$f(x)$	1	$\nearrow$	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
-
-

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	4
$f(x)$	1 ↗	2.2

- $P_n \; : \; 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4$
-

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$f(x) = \frac{3x-1}{x+1}$ $\qquad$ $f'(x) = \frac{4}{(x+1)^2}$

x	1	4
$f(x)$	1 ↗	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1		4
$f(x)$	1	↗	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

P_k vraie

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	4
$f(x)$	1 ↗	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

P_k vraie

$\implies 1 \leq u_{k+1} \leq u_k \leq 4$

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1			2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

P_k vraie

$\implies 1 \leq u_{k+1} \leq u_k \leq 4$

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$f(x) = \frac{3x-1}{x+1}$ $\qquad$ $f'(x) = \frac{4}{(x+1)^2}$

x	1	u_{k+1}	u_k	4
$f(x)$	1			2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
- P_k vraie

  $\implies 1 \leq u_{k+1} \leq u_k \leq 4$

  $\overset{f \nearrow}{\implies} f(1) \leq f\left(u_{k+1}\right) \leq f\left(u_k\right) \leq f(4)$

ex 28

$$\begin{cases} u_0 &= 4 \\ u_{n+1} &= f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

P_k vraie

$\implies 1 \leq u_{k+1} \leq u_k \leq 4$

$\overset{f \nearrow}{\implies} f(1) \leq f\left(u_{k+1}\right) \leq f\left(u_k\right) \leq f(4)$

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2{,}2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

P_k vraie

$\implies 1 \leq u_{k+1} \leq u_k \leq 4$

$\overset{f \nearrow}{\implies} f(1) \leq f\left(u_{k+1}\right) \leq f\left(u_k\right) \leq f(4)$

$\implies 1 \leq u_{k+2} \leq u_{k+1} \leq 2,2$

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2{,}2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

$$\begin{aligned} & P_k \text{ vraie} \\ \implies & 1 \leq u_{k+1} \leq u_k \leq 4 \\ \overset{f\nearrow}{\implies} & f(1) \leq f\left(u_{k+1}\right) \leq f\left(u_k\right) \leq f(4) \\ \implies & 1 \leq u_{k+2} \leq u_{k+1} \leq 2,2 \\ \implies & P_{k+1} \text{ vraie} \end{aligned}$$

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2{,}2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
- P_k vraie

  $\implies 1 \leq u_{k+1} \leq u_k \leq 4$

  $\overset{f \nearrow}{\implies} f(1) \leq f(u_{k+1}) \leq f(u_k) \leq f(4)$

  $\implies 1 \leq u_{k+2} \leq u_{k+1} \leq 2,2$

  $\implies P_{k+1}$ vraie
- Ccl : (u_n) décroissante minorée par 1

ex 28

$$\begin{cases} u_0 &= 4 \\ u_{n+1} &= f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
- $$\begin{aligned} & P_k \text{ vraie} \\ \implies & 1 \leq u_{k+1} \leq u_k \leq 4 \\ \overset{f \nearrow}{\implies} & f(1) \leq f(u_{k+1}) \leq f(u_k) \leq f(4) \\ \implies & 1 \leq u_{k+2} \leq u_{k+1} \leq 2,2 \\ \implies & P_{k+1} \text{ vraie} \end{aligned}$$
- Ccl : (u_n) décroissante minorée par 1 donc elle converge vers une limite ℓ

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2{,}2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

$$\begin{aligned} & P_k \text{ vraie} \\ \implies & 1 \leq u_{k+1} \leq u_k \leq 4 \\ \overset{f \nearrow}{\implies} & f(1) \leq f\left(u_{k+1}\right) \leq f\left(u_k\right) \leq f(4) \\ \implies & 1 \leq u_{k+2} \leq u_{k+1} \leq 2,2 \\ \implies & P_{k+1} \text{ vraie} \end{aligned}$$

- Ccl : (u_n) décroissante minorée par 1 donc elle converge vers une limite ℓ
- On "passe à la limite" dans la formule

$$u_{n+1} = \frac{3u_n - 1}{u_n + 1}$$

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f\left(u_n\right) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

$$\begin{aligned} & P_k \text{ vraie} \\ \implies & 1 \leq u_{k+1} \leq u_k \leq 4 \\ \overset{f \nearrow}{\implies} & f(1) \leq f(u_{k+1}) \leq f(u_k) \leq f(4) \\ \implies & 1 \leq u_{k+2} \leq u_{k+1} \leq 2,2 \\ \implies & P_{k+1} \text{ vraie} \end{aligned}$$

- Ccl : (u_n) décroissante minorée par 1 donc elle converge vers une limite ℓ
- On "passe à la limite" dans la formule

$$u_{n+1} = \frac{3u_n - 1}{u_n + 1} \implies \ell = \frac{3\ell - 1}{\ell + 1}$$

ex 28

$$\begin{cases} u_0 & = 4 \\ u_{n+1} & = f(u_n) = \frac{3u_n - 1}{u_n + 1} \end{cases}$$

$$f(x) = \frac{3x-1}{x+1} \qquad f'(x) = \frac{4}{(x+1)^2}$$

x	1	u_{k+1}	u_k	4
$f(x)$	1	u_{k+2}	u_{k+1}	2.2

- $P_n \ : \ 1 \leq u_{n+1} \leq u_n \leq 4$
- $1 \leq \underbrace{u_1}_{=2,2} \leq \underbrace{u_0}_{=4} \leq 4 \implies P_0$ vraie
-

$$\begin{aligned} & P_k \text{ vraie} \\ \implies & 1 \leq u_{k+1} \leq u_k \leq 4 \\ \overset{f \nearrow}{\implies} & f(1) \leq f(u_{k+1}) \leq f(u_k) \leq f(4) \\ \implies & 1 \leq u_{k+2} \leq u_{k+1} \leq 2,2 \\ \implies & P_{k+1} \text{ vraie} \end{aligned}$$

- Ccl : (u_n) décroissante minorée par 1 donc elle converge vers une limite ℓ
- On "passe à la limite" dans la formule

$$u_{n+1} = \frac{3u_n - 1}{u_n + 1} \implies \ell = \frac{3\ell - 1}{\ell + 1} \implies \ell = 1$$

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$
-
-
-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$
-
-
-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

-

-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} = \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n}$$

-

-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} = \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n}$$
  $$\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n}$$

-

-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$\begin{aligned} u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} &= \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n} \\ &\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} \\ &\leq 1 \end{aligned}$$

-

-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$\begin{aligned} u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} &= \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n} \\ &\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} \\ &\leq 1 \end{aligned}$$

- (u_n) croissante

-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$\begin{aligned} u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} &= \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n} \\ &\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} \\ &\leq 1 \end{aligned}$$

- (u_n) croissante *Rem* : $u_{n+1} - u_n$

-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$
- $$\begin{aligned} u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} &= \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n} \\ &\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} \\ &\leq 1 \end{aligned}$$
- (u_n) croissante *Rem* : $u_{n+1} - u_n = \sum_{k=1}^{n+1} \frac{1}{k \times 2^k} - \sum_{k=1}^{n} \frac{1}{k \times 2^k}$
-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$
- $$\begin{aligned} u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} &= \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n} \\ &\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} \\ &\leq 1 \end{aligned}$$
- (u_n) croissante *Rem* : $u_{n+1} - u_n = \sum_{k=1}^{n+1} \frac{1}{k \times 2^k} - \sum_{k=1}^{n} \frac{1}{k \times 2^k} = \frac{1}{(n+1) \times 2^{n+1}}$
-

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$\begin{aligned} u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} &= \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n} \\ &\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} \\ &\leq 1 \end{aligned}$$

- (u_n) croissante *Rem* : $u_{n+1} - u_n = \sum_{k=1}^{n+1} \frac{1}{k \times 2^k} - \sum_{k=1}^{n} \frac{1}{k \times 2^k} = \frac{1}{(n+1) \times 2^{n+1}}$

- (u_n) croissante majorée par 1

exemple du cours

- $$u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k}$$

- $$\begin{aligned} u_n = \sum_{k=1}^{n} \frac{1}{k \times 2^k} &= \frac{1}{1 \times 2^1} + \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} + \cdots + \frac{1}{n \times 2^n} \\ &\leq \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} \\ &\leq 1 \end{aligned}$$

- (u_n) croissante *Rem* : $u_{n+1} - u_n = \sum_{k=1}^{n+1} \frac{1}{k \times 2^k} - \sum_{k=1}^{n} \frac{1}{k \times 2^k} = \frac{1}{(n+1) \times 2^{n+1}}$

- (u_n) croissante majorée par 1 $\Longrightarrow$ (u_n) *converge*

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
-
-
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
-
-
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
-
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$H_{2n} - H_n$$
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$H_{2n} - H_n = \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k}$$
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \end{aligned}$$
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \end{aligned}$$
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
-
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ;
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n} \right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} \right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ; ou bien elle ne l'est pas, et alors $H_n \longrightarrow +\infty$
-
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ; ou bien elle ne l'est pas, et alors $H_n \longrightarrow +\infty$
- Par l'absurde : si $H_n \longrightarrow \ell$, alors
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ; ou bien elle ne l'est pas, et alors $H_n \longrightarrow +\infty$
- Par l'absurde : si $H_n \longrightarrow \ell$, alors $H_{2n} \longrightarrow \ell$
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ; ou bien elle ne l'est pas, et alors $H_n \longrightarrow +\infty$
- Par l'absurde : si $H_n \longrightarrow \ell$, alors $H_{2n} \longrightarrow \ell$
$$H_{2n} - H_n \geq \frac{1}{2}$$
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ; ou bien elle ne l'est pas, et alors $H_n \longrightarrow +\infty$
- Par l'absurde : si $H_n \longrightarrow \ell$, alors $H_{2n} \longrightarrow \ell$

  $$H_{2n} - H_n \geq \frac{1}{2} \implies \ell - \ell \geq \frac{1}{2}$$
-

ex 31

- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ; ou bien elle ne l'est pas, et alors $H_n \longrightarrow +\infty$
- Par l'absurde : si $H_n \longrightarrow \ell$, alors $H_{2n} \longrightarrow \ell$
  $$H_{2n} - H_n \geq \frac{1}{2} \implies \ell - \ell \geq \frac{1}{2} \implies 0 \geq \frac{1}{2}$$
-

ex 31

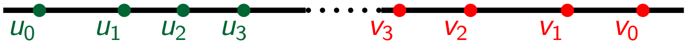
- $$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad \text{(série harmonique)}$$
- (H_n) croissante
- $$\begin{aligned} H_{2n} - H_n &= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k} \\ &= \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}\right) - \left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right) \\ &= \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \end{aligned}$$
- Deux alternatives : ou bien (H_n) est majorée, et alors elle converge ; ou bien elle ne l'est pas, et alors $H_n \longrightarrow +\infty$
- Par l'absurde : si $H_n \longrightarrow \ell$, alors $H_{2n} \longrightarrow \ell$
  $$H_{2n} - H_n \geq \frac{1}{2} \implies \ell - \ell \geq \frac{1}{2} \implies 0 \geq \frac{1}{2}$$
- Ccl : $H_n \longrightarrow +\infty$

suites adjacentes

- Deux suites $(u_n)_{n\in\mathbb{N}}$ et $(v_n)_{n\in\mathbb{N}}$ sont dites adjacentes si :
  - $(u_n)_{n\in\mathbb{N}}$ est croissante,
  - $(v_n)_{n\in\mathbb{N}}$ est décroissante,
  - $v_n - u_n \to 0$.

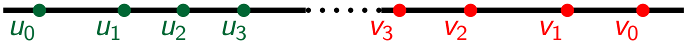
suites adjacentes

- Deux suites $(u_n)_{n\in\mathbb{N}}$ et $(v_n)_{n\in\mathbb{N}}$ sont dites adjacentes si :
  - $(u_n)_{n\in\mathbb{N}}$ est croissante,
  - $(v_n)_{n\in\mathbb{N}}$ est décroissante,
  - $v_n - u_n \to 0$.



suites adjacentes

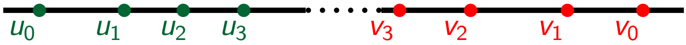
- Deux suites $(u_n)_{n\in\mathbb{N}}$ et $(v_n)_{n\in\mathbb{N}}$ sont dites adjacentes si :
  - $(u_n)_{n\in\mathbb{N}}$ est croissante,
  - $(v_n)_{n\in\mathbb{N}}$ est décroissante,
  - $v_n - u_n \to 0$.



- Théorème : deux suites adjacentes convergent vers la même limite.

suites adjacentes

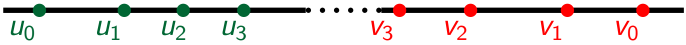
- Deux suites $(u_n)_{n\in\mathbb{N}}$ et $(v_n)_{n\in\mathbb{N}}$ sont dites adjacentes si :
  - $(u_n)_{n\in\mathbb{N}}$ est croissante,
  - $(v_n)_{n\in\mathbb{N}}$ est décroissante,
  - $v_n - u_n \to 0$.



- Théorème : deux suites adjacentes convergent vers la même limite.
- exple du DM9 :

suites adjacentes

- Deux suites $(u_n)_{n\in\mathbb{N}}$ et $(v_n)_{n\in\mathbb{N}}$ sont dites adjacentes si :
  - $(u_n)_{n\in\mathbb{N}}$ est croissante,
  - $(v_n)_{n\in\mathbb{N}}$ est décroissante,
  - $v_n - u_n \to 0$.

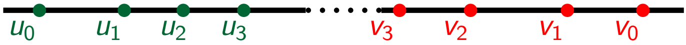


- Théorème : deux suites adjacentes convergent vers la même limite.
- exple du DM9 :

$$u_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} - 2\sqrt{n} \qquad \text{et} \qquad v_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} - 2\sqrt{n+1}$$

suites adjacentes

- Deux suites $(u_n)_{n\in\mathbb{N}}$ et $(v_n)_{n\in\mathbb{N}}$ sont dites adjacentes si :
  - $(u_n)_{n\in\mathbb{N}}$ est croissante,
  - $(v_n)_{n\in\mathbb{N}}$ est décroissante,
  - $v_n - u_n \to 0$.



- Théorème : deux suites adjacentes convergent vers la même limite.
- exple du DM9 :

$$u_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} - 2\sqrt{n} \qquad \text{et} \qquad v_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} - 2\sqrt{n+1}$$

relations de comparaison

Définitions :

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n \in \mathbb{N}}$ bornée

Exemples :

- $n + 1 \sim n$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n + 1 \sim n$ $\qquad \frac{n+1}{n}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n + 1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $\mathrm{e}^n = o\left(\mathrm{e}^{2n}\right)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n \in \mathbb{N}}$ bornée

Exemples :

- $n + 1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $\mathrm{e}^n = o\left(\mathrm{e}^{2n}\right) \qquad \frac{\mathrm{e}^n}{\mathrm{e}^{2n}} = \mathrm{e}^{-n}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n + 1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $\mathrm{e}^n = o\left(\mathrm{e}^{2n}\right) \qquad \frac{\mathrm{e}^n}{\mathrm{e}^{2n}} = \mathrm{e}^{-n} \longrightarrow 0$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $\mathrm{e}^n = o\left(\mathrm{e}^{2n}\right) \qquad \frac{\mathrm{e}^n}{\mathrm{e}^{2n}} = \mathrm{e}^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $\mathrm{e}^n = o\left(\mathrm{e}^{2n}\right) \qquad \frac{\mathrm{e}^n}{\mathrm{e}^{2n}} = \mathrm{e}^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $\mathrm{e}^n = o\left(\mathrm{e}^{2n}\right) \qquad \frac{\mathrm{e}^n}{\mathrm{e}^{2n}} = \mathrm{e}^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

- $n + 1 = o\left(n^2 + 3\right)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right)$ $\quad \frac{n+1}{n^2+3}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \Longrightarrow \frac{u_n}{v_n} \to 0 \Longrightarrow u_n = O(v_n)$
- $u_n \sim v_n \Longrightarrow \frac{u_n}{v_n} \to 1 \Longrightarrow u_n = O(v_n)$

Autre exemples :

- $n+1 = o\left(n^2+3\right)$ $\quad \frac{n+1}{n^2+3} \longrightarrow 0$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

- $n+1 = o\left(n^2+3\right)$ $\quad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right)$ $\quad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1)$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right)$ $\quad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right)$ $\quad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$ $\implies$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} =$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff (u_n)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$
  $\implies u_n \times v_n \sim a_n \times b_n$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n$ $\quad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right)$ $\quad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n)$ $\quad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right)$ $\quad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$
  $\implies u_n \times v_n \sim a_n \times b_n$
- $n! \sim$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $\mathrm{e}^n = o\left(\mathrm{e}^{2n}\right) \qquad \frac{\mathrm{e}^n}{\mathrm{e}^{2n}} = \mathrm{e}^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$
  $\implies u_n \times v_n \sim a_n \times b_n$
- $n! \sim \sqrt{2\pi n}\left(\frac{n}{\mathrm{e}}\right)^n$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$
  $\implies u_n \times v_n \sim a_n \times b_n$
- $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \implies \binom{2n}{n} =$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$
  $\implies u_n \times v_n \sim a_n \times b_n$
- $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \implies \binom{2n}{n} = \frac{(2n)!}{n! \times n!} \sim$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o(v_n) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O(v_n) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o(v_n) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O(v_n)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O(v_n)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff (u_n)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$
  $\implies u_n \times v_n \sim a_n \times b_n$
- $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \implies \binom{2n}{n} = \frac{(2n)!}{n! \times n!} \sim \frac{4^n}{\sqrt{n\pi}}$

relations de comparaison

Définitions :

1. $u_n \sim v_n \iff \frac{u_n}{v_n} \to 1$
2. $u_n = o\left(v_n\right) \iff \frac{u_n}{v_n} \to 0$
3. $u_n = O\left(v_n\right) \iff \left(\frac{u_n}{v_n}\right)_{n\in\mathbb{N}}$ bornée

Exemples :

- $n+1 \sim n \qquad \frac{n+1}{n} = 1 + \frac{1}{n} \longrightarrow 1$
- $e^n = o\left(e^{2n}\right) \qquad \frac{e^n}{e^{2n}} = e^{-n} \longrightarrow 0$
- $\ln n = o(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ (C.C.)
- $\ln n = O(n) \qquad \frac{\ln n}{n} \longrightarrow 0$ et toute suite cv est bornée
- $u_n = o\left(v_n\right) \implies \frac{u_n}{v_n} \to 0 \implies u_n = O\left(v_n\right)$
- $u_n \sim v_n \implies \frac{u_n}{v_n} \to 1 \implies u_n = O\left(v_n\right)$

Autre exemples :

- $n+1 = o\left(n^2+3\right) \qquad \frac{n+1}{n^2+3} \longrightarrow 0$
- $\ln(2n) \sim \ln n$
  $\frac{\ln(2n)}{\ln n} = \frac{\ln 2 + \ln n}{\ln n} = \frac{\ln 2}{\ln n} + 1 \longrightarrow 1$
- $u_n = o(1) \iff \frac{u_n}{1} \longrightarrow 0 \iff u_n \longrightarrow 0$
- $u_n = O(1) \iff \left(\frac{u_n}{1}\right)_{n\in\mathbb{N}}$ bornée $\iff \left(u_n\right)_{n\in\mathbb{N}}$ bornée
- $u_n \sim a_n$ et $v_n \sim b_n$
  $\implies \frac{u_n}{a_n} \longrightarrow 1$ et $\frac{v_n}{b_n} \longrightarrow 1$
  $\implies \frac{u_n \times v_n}{a_n \times b_n} = \frac{u_n}{a_n} \times \frac{v_n}{b_n} \longrightarrow 1$
  $\implies u_n \times v_n \sim a_n \times b_n$
- $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \implies \binom{2n}{n} = \frac{(2n)!}{n! \times n!} \sim \frac{4^n}{\sqrt{n\pi}}$